

HW 3 Section 3.5 # 6, 24, 46, 58, 62

6. The matrix of cofactors is

$$\begin{bmatrix} \begin{vmatrix} 2 & 3 \\ -1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ -1 & -2 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}.$$

So, the adjoint of  $A$  is

$$\text{adj}(A) = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}.$$

Because  $\det(A) = 0$ , the matrix  $A$  has no inverse.

24. The coefficient matrix is

$$A = \begin{bmatrix} -0.4 & 0.8 \\ 0.2 & 0.3 \end{bmatrix}, \quad \text{so} \quad |A| = -0.28.$$

Because  $|A| \neq 0$ , we can use Cramer's Rule.

$$A_1 = \begin{bmatrix} 1.6 & 0.8 \\ 0.6 & 0.3 \end{bmatrix}, \quad |A_1| = 0$$

$$A_2 = \begin{bmatrix} -0.4 & 1.6 \\ 0.2 & 0.6 \end{bmatrix}, \quad |A_2| = -0.56$$

The solution is

$$x_1 = \frac{|A_1|}{|A|} = \frac{0}{-0.28} = 0 \quad x_2 = \frac{|A_2|}{|A|} = \frac{-0.56}{-0.28} = 2.$$

46.

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 4 & 2 & 1 \end{vmatrix} = \pm \frac{1}{2}(-8) = 4.$$

58.

$$\begin{aligned} \text{Volume} &= \pm \frac{1}{6} \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} \\ &= \pm \frac{1}{6} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & -1 & 1 \\ -1 & 1 & 2 & 1 \end{vmatrix} = \frac{1}{6}(3) = \frac{1}{2} \end{aligned}$$

62. Since

$$\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & -2 & -5 & 1 \\ 2 & 6 & 11 & 1 \end{vmatrix} = 0$$

the four points are coplanar.